EPITA

S1 S2

Mathematics

The basic functions

Presentation

This documents provides an overview of the basic functions. You have to know them by heart!

Keep it available during the whole year... and during next year too!

For each function, the document provides the graph, algebraic properties, reference values, etc.

Table des matières

1 The absolute value function **2**

2 The square function **3**

3 The square root function **4**

4 The inverse function **5**

5 The exponential function **6**

6 The logarithm function **7**

7 The sine and cosine functions **8**

8 The tangent function **9**

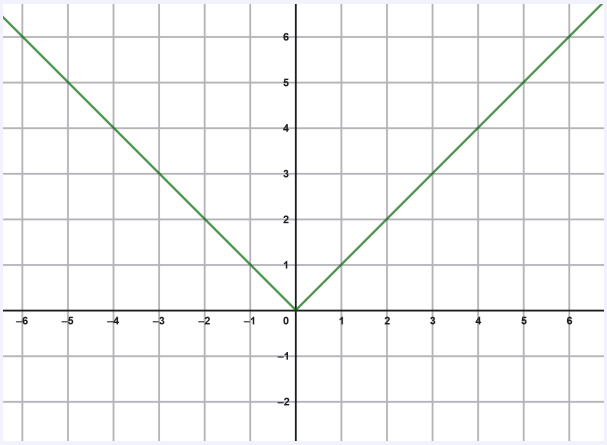
1 The absolute value function

Presentation

The **absolute value** function is defined for all $x \in \mathbb{R}$ by $f(x) = |x|$.

That is, $f(x) = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases}$

Its graph is the following:



Properties

1. $\forall\, a \in \mathbb{R}, \ |-a| = |a|$ (the function is even)
2. $\forall\, a \in \mathbb{R}, \ |a| = 0 \iff a = 0$
3. $\forall\, (a, b) \in \mathbb{R}^2, \ |a \times b| = |a| \times |b|$ and, if $b \neq 0$, $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$



$|a + b| \neq |a| + |b|$.

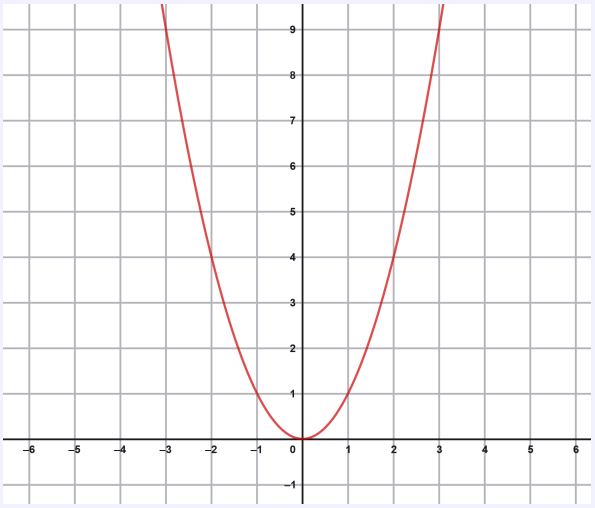
However, for all real numbers a and b, $|a + b| \leq |a| + |b|$ (triangle inequality)!!

2 The square function

Presentation

The **square** function is defined for all $x \in \mathbb{R}$ by $f(x) = x^2$.

Its graph is the following:



Properties

1. $\forall a \in \mathbb{R}$, $(-a)^2 = a^2$ (the function is even)
2. $\forall a \in \mathbb{R}$, $a^2 = 0 \iff a = 0$
3. $\forall (a, b) \in \mathbb{R}^2$, $(a \times b)^2 = a^2 \times b^2$ and, if $b \neq 0$, $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$



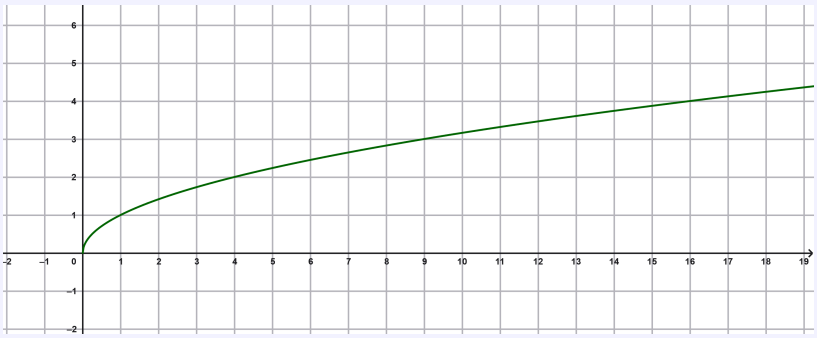
$(a + b)^2 \neq a^2 + b^2$!!

3 The square root function

Presentation

The square root of a positive real number a is the positive real number whose square equals a.
It is denoted by $\sqrt{a}$. Hence, for all positive real number, $(\sqrt{a})^2 = a$
The **square root** function is defined for all $x \in \mathbb{R}^+ = [0, +\infty[$ by $f(x) = \sqrt{x}$.

Its graph is the following:



Properties

1. $\forall\, a \in \mathbb{R}^+$, $(\sqrt{a})^2 = a$
2. $\forall\, a \in \mathbb{R}$, $\sqrt{a^2} = |a|$
3. $\forall\, (a, b) \in (\mathbb{R}^+)^2$, $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$ and, if $b \neq 0$, $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$



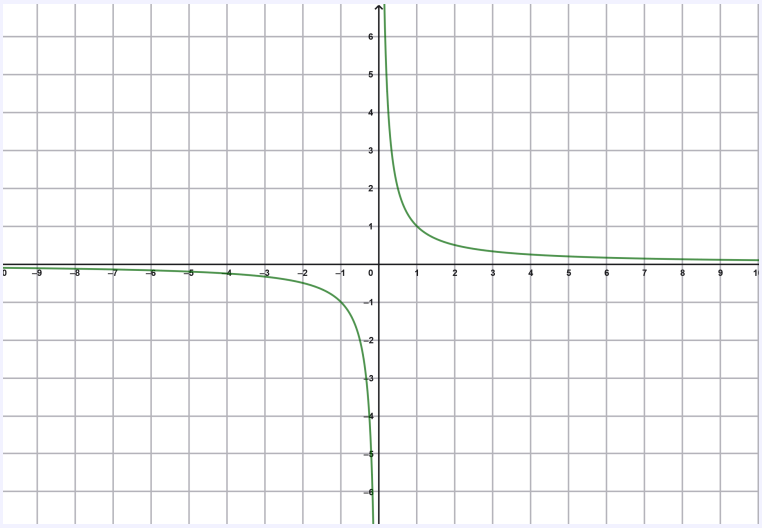
$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$!!

4 The inverse function

Presentation

The **inverse** function is defined for all $x \in \mathbb{R}^* =]-\infty, 0[\cup]0, +\infty[$ by $f(x) = \dfrac{1}{x}$.

Its graph is the following:



Properties

1. $\lim\limits_{x \to +\infty} \dfrac{1}{x} = 0^+$ and $\lim\limits_{x \to -\infty} \dfrac{1}{x} = 0^-$
2. $\lim\limits_{x \to 0^+} \dfrac{1}{x} = +\infty$ and $\lim\limits_{x \to 0^-} \dfrac{1}{x} = -\infty$



$\dfrac{1}{a+b} \neq \dfrac{1}{a} + \dfrac{1}{b}$!!

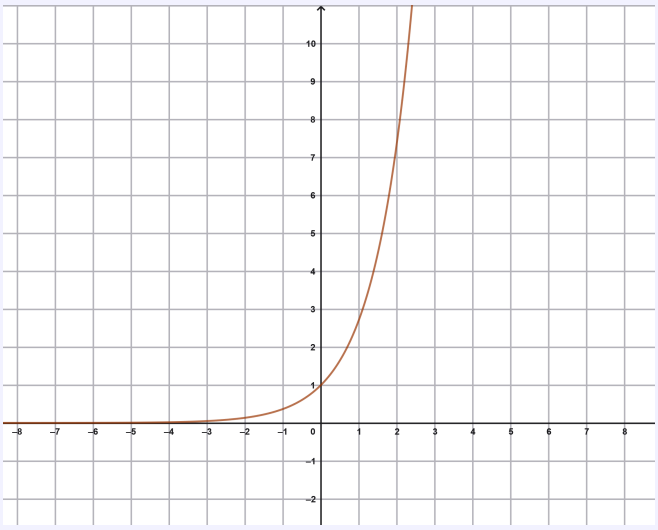
5 The exponential function

Presentation

The exponential function is the unique function which is differentiable on $\mathbb{R}$, equal to its derivative and which takes value 1 at $x = 0$.

The **exponential** function is defined for all $x \in \mathbb{R}$ by $f(x) = e^x$.

Its graph is the following:



Properties

1. $e^0 = 1$ and e^1 is denoted by e.
2. $\forall\, a \in \mathbb{R},\ e^a > 0$
3. $\forall\, (a, b) \in \mathbb{R}^2,\ e^{a+b} = e^a \times e^b$
4. $\forall\, a \in \mathbb{R},\ e^{-a} = \dfrac{1}{e^a}$ and $e^{a-b} = \dfrac{e^a}{e^b}$
5. $\forall\, (a, n) \in \mathbb{R} \times \mathbb{Z},\ (e^a)^n = e^{na}$



This function admits no root: no value $x \in \mathbb{R}$ has a zero exponential!!

6 The logarithm function

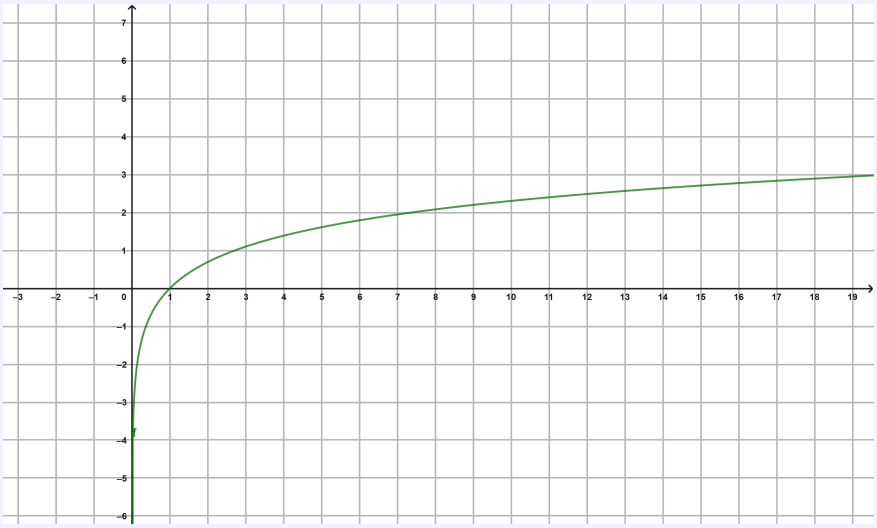
Presentation

The natural logarithm of a strictly positive real number a is the unique solution of the equation $e^x = a$ of variable x. It is denoted by $x = \ln(a)$.

The **natural logarithm** function, denoted by ln, is the function

$$\ln : \left\{ \begin{array}{ccc}]0,+\infty[& \longrightarrow & \mathbb{R} \\ x & \longmapsto & \ln(x) \end{array} \right.$$

Its graph is the following:



Properties

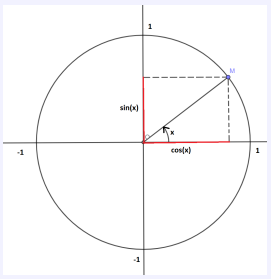
1. $\forall\, x > 0$ and $\forall\, y \in \mathbb{R}$ $\quad (e^y = x \iff y = \ln(x))$
2. $\ln(1) = 0$ and $\ln(e) = 1$
3. $\forall\, x \in \mathbb{R}$, $\ln(e^x) = x$ and $\forall\, x \in \mathbb{R}^{+*}$, $e^{\ln(x)} = x$
4. $\forall\, a \in]0,1[$, $\ln(a) < 0$ and $\forall\, a > 1$, $\ln(a) > 0$
5. $\forall\, (a,b) \in]0,+\infty[^2$, $\ln(a \times b) = \ln(a) + \ln(b)$
6. $\forall\, a \in \mathbb{R}^{+*}$, $\ln\left(\dfrac{1}{a}\right) = -\ln(a)$ and $\forall\, (a,b) \in (\mathbb{R}^{+*})^2$, $\ln\left(\dfrac{a}{b}\right) = \ln(a) - \ln(b)$
7. $\forall\, (a,n) \in \mathbb{R} \times \mathbb{Z}$, $\ln(a^n) = n \ln(a)$



$\ln(a+b) \neq \ln(a) \times \ln(b)$

7 The sine and cosine functions

Trigonometric circle



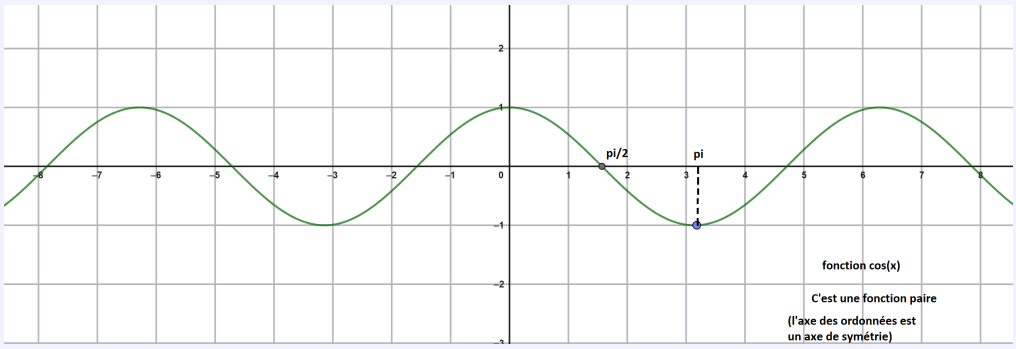
Properties and some reference values

angle x (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

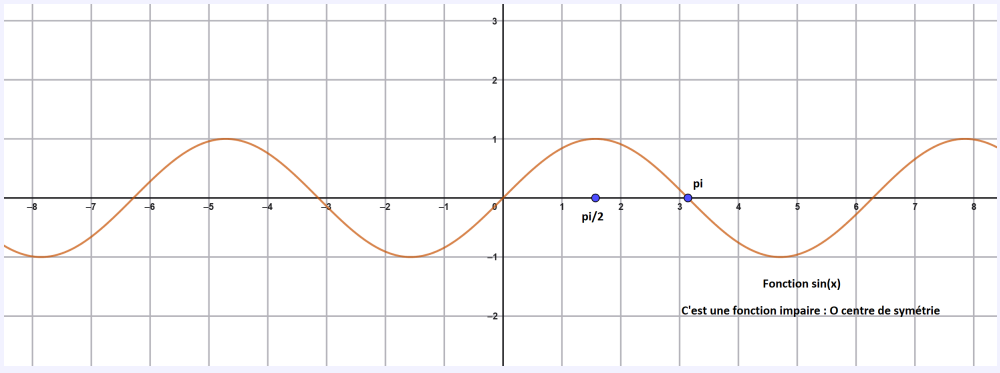
The cosine function is even, the sine function is odd:

$$\forall x \in \mathbb{R}, \quad \cos(-x) = \cos(x) \quad \text{and} \quad \sin(-x) = -\sin(x)$$

Graph of the cosine function



Graph of the sine function



8 The tangent function

Presentation

The **tangent** function id $\tan : x \longmapsto \tan(x) = \dfrac{\sin(x)}{\cos(x)}$.

It is defined on $\mathbb{R} \setminus \left\{ \dfrac{\pi}{2} + k\pi,\ k \in \mathbb{Z} \right\}$.

Its graph is the following:

